

Senior  
UAE Math modeling

MA 465 Introduction to Mathematical Modeling  
Senior Level Course

This course is a special offering of MA465 in which mathematical models of components of the Earth System are used to illustrate concepts of mathematical model building and analysis. Examples will include models of the atmosphere, oceans and biological populations. The course is designed as a project course in which emphasis is placed on two team projects involving the building, solution and interpretation of a mathematical model. The first model topic can be chosen by the modeling team. For the second project, all teams will be asked to build a simple model of the Earth's climate system and addressing global warming and whether CO2 controls should be implemented.

**Week 1**

Concepts of Mathematical Modeling -translating problems into a mathematical framework, simplification, scaling and testing.

As motivation for this special course a general introduction is given of the Earth's Climate system as a system worthy of modeling due to its complexity, importance and mathematical interest.

**Week 2**

A model of a cooling coffee cup is used as an example of translating physical concepts such as radiative cooling, evaporative cooling, convective cooling etc into mathematical expressions. This is then extended into developing an energy budget model for the ground surface.

Simple radiative equilibrium models of the Earth are developed

**Week 3**

Dimensional analysis is introduced including the Buckingham Pi Theorem

Dimensional analysis is used to develop a model of the wind profile near the ground surface. Concept of friction velocity is derived from the analysis.

**Week 4**

A model of heat conduction into the soil driven by diurnal heating at the surface is developed and solve analytically.

Models of biological population models are introduced including the Malthusian models, the logistic model and Lotka-Volterra models.

#### **Week 5**

A population model of whales and krill in the Antarctic is developed and analysed as a dynamical system. Concepts of solution behavior such as limit cycles, stable attractors etc. are explained in terms of the characteristic equation. Also, the discrete problem is introduced in and analysed as a matrix power problem- the physical meaning of eigenvalues is discussed.

Mid-term Exam

#### **Week 6**

First Project Presentation

Simple numerical techniques are introduced for solving models involving set of ordinary differential equations. The whale / krill nonlinear problem is used as an example.

#### **Week 7**

The concept of population behavior for phytoplankton and food chain behavior in the ocean is discussed in terms of limiting nutrients.

A simple ocean model for wind driven coastal upwelling is developed. The governing Navier-Stokes equations are simply written down and terms explained. Simplifications in terms of reduction in physical dimension and linearization are introduced to the point that the problem is reduced to a second order ordinary differential equation for which solutions are obtained.

#### **Week 8**

The solution and the role of upwelling in bringing nutrients into the euphotic zone and affecting population dynamics is discussed.

A general discussion of the physics of the Earth climate system is given as preparation for the second project. Concepts of positive and negative feedback

is introduced and the mathematical structure for incorporating this feedback is described.

#### **Week 9**

A simple model for the dispersion of pollutants in the atmosphere is developed. The model is developed from an initial PDE diffusion equation. The PDE is quickly reduced to an ODE model and analytical solutions are obtained in terms of a Gaussian dispersion kernel. The model is applied to point sources of pollution such as power plants or volcanoes. The wind profile model developed the third week of class is incorporated showing how models can be pieced together. Statistical techniques in terms of density functions are introduced to give pollution estimates on longer time scales.

#### **Week 10**

Second Project Presentation

Final Exam