

Senior
UAE Math modeling

MA 465 Introduction to Mathematical Modeling
Senior Level Course

This course is a special offering of MA465 in which mathematical models of components of the Earth System are used to illustrate concepts of mathematical model building and analysis. Examples will include models of the atmosphere, oceans and biological populations. The course is designed as a project course in which emphasis is placed on two team projects involving the building, solution and interpretation of a mathematical model. The first model topic can be chosen by the modeling team. For the second project, all teams will be asked to build a simple model of the Earth's climate system and addressing global warming and whether CO2 controls should be implemented.

Week 1

Concepts of Mathematical Modeling -translating problems into a mathematical framework, simplification, scaling and testing.

As motivation for this special course a general introduction is given of the Earth's Climate system as a system worthy of modeling due to its complexity, importance and mathematical interest.

Week 2

A model of a cooling coffee cup is used as an example of translating physical concepts such as radiative cooling, evaporative cooling, convective cooling etc into mathematical expressions. This is then extended into developing an energy budget model for the ground surface.

Simple radiative equilibrium models of the Earth are developed

Week 3

Dimensional analysis is introduced including the Buckingham Pi Theorem

Dimensional analysis is used to develop a model of the wind profile near the ground surface. Concept of friction velocity is derived from the analysis.

Week 4

A model of heat conduction into the soil driven by diurnal heating at the surface is developed and solve analytically.

Models of biological population models are introduced including the Malthusian models, the logistic model and Lotka-Volterra models.

Week 5

A population model of whales and krill in the Antarctic is developed and analysed as a dynamical system. Concepts of solution behavior such as limit cycles, stable attractors etc. are explained in terms of the characteristic equation. Also, the discrete problem is introduced in and analysed as a matrix power problem- the physical meaning of eigenvalues is discussed.

Mid-term Exam

Week 6

First Project Presentation

Simple numerical techniques are introduced for solving models involving set of ordinary differential equations. The whale / krill nonlinear problem is used as an example.

Week 7

The concept of population behavior for phytoplankton and food chain behavior in the ocean is discussed in terms of limiting nutrients.

A simple ocean model for wind driven coastal upwelling is developed. The governing Navier-Stokes equations are simply written down and terms explained. Simplifications in terms of reduction in physical dimension and linearization are introduced to the point that the problem is reduced to a second order ordinary differential equation for which solutions are obtained.

Week 8

The solution and the role of upwelling in bringing nutrients into the euphotic zone and affecting population dynamics is discussed.

A general discussion of the physics of the Earth climate system is given as preparation for the second project. Concepts of positive and negative feedback

is introduced and the mathematical structure for incorporating this feedback is described.

Week 9

A simple model for the dispersion of pollutants in the atmosphere is developed. The model is developed from an initial PDE diffusion equation. The PDE is quickly reduced to an ODE model and analytical solutions are obtained in terms of a Gaussian dispersion kernel. The model is applied to point sources of pollution such as power plants or volcanoes. The wind profile model developed the third week of class is incorporated showing how models can be pieced together. Statistical techniques in terms of density functions are introduced to give pollution estimates on longer time scales.

Week 10

Second Project Presentation

Final Exam

Attachment: Course Description and Syllabus

Introduction to Mathematical Modeling (MA465)

(Formerly MA425) Applying mathematics by formulating, analyzing, and criticizing mathematical models of various phenomena. Examples will be chosen from the physical, biological, and social sciences. Emphasizes development and use of simple mathematical models by having students study general modeling principles and case studies (some open-ended) drawn from various sources.

This course has been taught in the Mathematics Department for approximately five years. It is intended that the course expose senior level mathematicians and science majors to the concepts of mathematical modeling. An emphasis is placed on taking a complicated physical or biological system, simplifying the system through assumptions, and describing the components mathematically then solving a resulting system of low order equations. In the past, examples of a general nature have been used including spring and mass systems, population models, traffic control models, and economic models. Under the ESSE program, all examples covered will be parts of or simplifications to the Earth system.

During this past year as preparation for the ESSE program some Earth system examples were taught in the Fall version of the course. These included simple energy balance climate models, katabatic wind models, and air pollution dispersion models. In addition, a graduate level modeling course was taught centered around building ocean, atmosphere and ice models of the Antarctic.

Next Spring, we plan to offer the MA465 - Introduction to Mathematical Modeling as the ESSE course during the 1994 Spring term. In the past this course has attracted students from physics, chemistry, and biology, but, the majority have been mathematics majors. We intend to place special announcements in biology, chemistry, physics and computer sciences to draw a broader mix to this special offering.

Because this is a mathematics course and mathematics through differential equations is required, students should be capable of tackling fairly sophisticated models. Some students will have had some partial differential equations.

The following lists a simplified syllabus.

1. Basic Concepts in Mathematical Modeling
 - a. Assumptions
 - b. Dimensional Analysis
2. Energy Balance Models
 - a. Formulation
 - b. Solution as root-finding technique
3. Simplified Wind Models/Ocean Models/Ice Models
 - a. Katabatic flows
 - b. Two-layer ocean
 - c. Polynya models
4. Simple Population Models
 - a. Single species
 - b. Predator-prey
 - c. Concepts of nonlinear dynamics
5. Models of the Earth System
 - a. PDE models
 - b. ODE - Non-linear systems

The course will emphasize team projects. Two projects will have to be developed and presented.